ESTIMATING THE CORRELATION OF NON-CONTEMPORANEOUS TIME-SERIES: AN OVERVIEW[1]

DRAFT

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A common task in applied finance is measuring the correlation between returns for two series, but daily financial time-series are often observed at different times: for example the FTSE stock index "close" is about 11am NY time while the S&P 500 stock index closes about 4pm NY time. In such a case the naïve correlation estimator is biased, often substantially so. This paper discusses strategies for estimating the correlation in the face of such non-contemporaneous observations. A detailed discussion of the problem is in a companion paper, Coleman (2007) on the web at http://ssrn.com/abstract=987119.

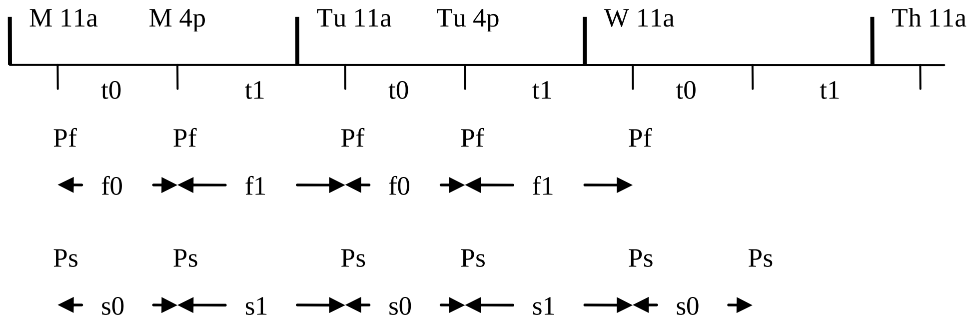
I focus on the case where series are serially independent and normally distributed. Even in this idealized case estimation can be challenging. Building on earlier work of Robb (1987) and Kahya (1998) I propose a pseudo-maximum likelihood estimator and two, much simpler, method-of-moment estimators. I compare these using simulations and derive some approximate expressions for standard errors (relative to the contemporaneous-data case).

One important conclusion from the simulations is that the standard error of the correlation estimator can be quite high relative to the contemporaneous observation case. In contrast the standard error for the covariance is not dramatically higher. The inference seems to be that one can obtain reasonably good (moderate standard-error) estimates of the covariance even though data are non-contemporaneous, but one cannot obtain <u>joint</u> information about the covariance and variances with the same degree of confidence. The problem seems to be that the non-overlapping data introduce enough random variation that one has difficulty estimating variances and covariances jointly, although one can determine each separately.

[1] I would like to thank John Teall for help and advice, and Sandy Grossman for pointing out the non-trivial nature of this problem. Errors are my own. This paper is available for download at www.hilerun.org/tsc/noncontemp_mini.pdf. The detailed companion paper is available for download at http://papers.ssrn.com/abstract=987119

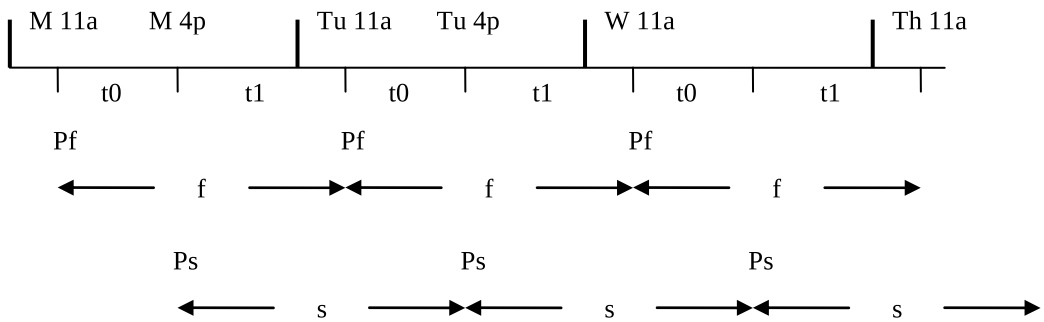
Consider the FTSE and S&P indexes. If we assume that prices are observed as contemporaneous "snap-shots" at both 11am and 4pm (i.e. twice per day as shown in the diagram below) there would be no problem. We would simply have returns over unequal time-periods (five hours and 19 hours) but we could easily implement a maximum likelihood estimator.

Contemporaneous Sampling Scheme



The sampling scheme above does not apply since in reality we cannot get a price snap-shot for the FTSE at 4pm NY time. So we have the non-contemporaneous but overlapping sampling scheme as outlined below: the FTSE return for Wednesday (closing Wednesday 11am) overlaps both the Tuesday and the Wednesday S&P return. This does, however, point us in the direction of a simple set of estimators, the method-of-moment (MoM) estimators.

Non-Contemporaneous Sampling Scheme



Naïve and Method-of-Moments Estimators

The observed returns are partially overlapping but they are composed of (unobserved) pieces that are alternately overlapping and not. Treating the observed returns as composed of unobserved pieces one can write the variance of the returns x as:

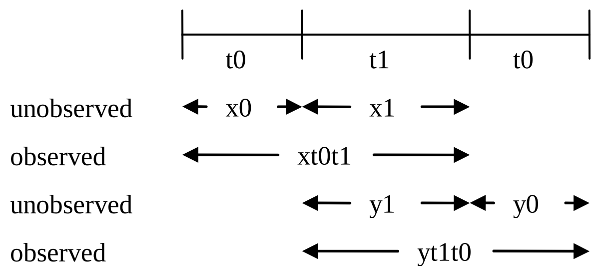
$$V(x) = V(x_0) + V(x_1) = t_0 \cdot \sigma_x^2 + t_1 \cdot \sigma_x^2$$

Similarly the variance of y will be:

$$V(y) = V(y_1) + V(y_0) = t_1 \cdot \sigma_y^2 + t_0 \cdot \sigma_y^2 \ .$$

Most importantly, consider the covariance across the t_1-overlap data:

t_1-overlap data ("same-day" data)



$$Cov(x_{t0t1},y_{t1t0}) = Cov(x_0,y_1) + Cov(x_0,y_0) + Cov(x_1,y_1) + Cov(x_1,y_0)$$

with all except (x_1,y_1) being independent. This means

$$Cov(x_{t0t1},y_{t1t0}) = t_1 \cdot \sigma_{xy} / \sigma_x \cdot \sigma_y$$

(presuming that $t_1+t_0=1$; otherwise the factor $1/(t_1+t_0)$ will appear). The population correlation coefficient will be

$$\rho \equiv \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{Cov\left(x_{t0t1}, y_{t1t0}\right)}{\sqrt{Var\left(x_{t0t1}\right) \cdot Var\left(y_{t0t1}\right)}} / t_1 \quad .$$

The naïve correlation estimator (the ratio of sample covariance to variances) is biased in that it converges in probability to $t_1 \cdot \rho$: [2]

$$p\lim\left[\frac{\widehat{Cov}\left(x_{t0t1}, y_{t1t0}\right)}{\sqrt{\widehat{Var}\left(x_{t0t1}\right) \cdot \widehat{Var}\left(y_{t0t1}\right)}}\right] = t_1 \cdot \frac{\sigma_{xy}}{\sigma_x \sigma_y} = t_1 \cdot \rho \quad .$$

The bias can be removed simply by dividing by t_1 (assuming $(t_1+t_0)=1$):

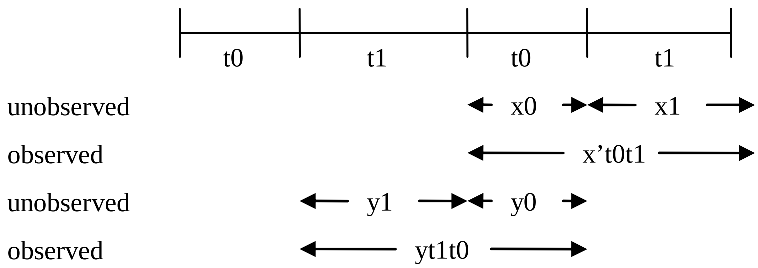
(1a) $$\hat{\rho}^{1,mms} = \frac{\widehat{Cov}\left(x_{t0t1}, y_{t1t0}\right)}{\sqrt{\widehat{Var}\left(x_{t0t1}\right) \cdot \widehat{Var}\left(y_{t0t1}\right)}} / t_1 \quad .$$

[2] This presumes $t_1+t_0=1$. If not the bias will be $t_1/(t_1+t_0)$.

We call this the method-of-moments t_1-separate estimator because it replaces population by sample moments and uses t_1-overlap data. Asymptotically this estimator will be consistent but in small samples may be outside the range [-1,1].

One could also use t_0-overlap data:

t_0-overlap data ("y-lagged" data)



$$Cov(x'_{t0t1},y_{t1t0}) \;=\; Cov(x_0,y_1) + Cov(x_0,y_0) + Cov(x_1,y_1) + Cov(x_1,y_0)$$

Here all except (x_0,y_0) are independent. This means

$$Cov(x'_{t0t1},y_{t1t0}) \;=\; t_0\cdot\sigma_{xy} \,/\, \sigma_x\cdot\sigma_y$$

and leads directly to a method-of-moments t_0-separate estimator for the correlation as:

(1b) $$\hat{\rho}^{0,mms} = \frac{\widehat{Cov}\left(x'_{t0t1}, y_{t1t0}\right)}{\sqrt{\widehat{Var}\left(x'_{t0t1}\right)\cdot\widehat{Var}\left(y_{t0t1}\right)}} \,/\, t_0$$

This provides a second consistent estimator of the correlation but once again for small samples it may be outside the interval [-1,1].

We can combine the two estimators to give a method-of-moments combined estimator, by taking a weighted average (essentially as Kahya 1998 does):

(1c) $$\hat{\rho}^{mmc} = \left(\hat{\rho}^{1,mms}\cdot t_1 + \hat{\rho}^{0,mms}\cdot t_0\right)/\left(t_1 + t_0\right) \; .$$

For small samples this actually produces two estimators, depending on whether one applies the restriction $-1\leq\rho\leq1$ to the separate estimates before averaging or to the combined estimator after averaging. Simulation results show that the better choice is the latter, applying the restriction to the combined estimator after averaging.

A natural way to estimate parameters such as the correlation coefficient is by maximum likelihood. Unfortunately, for overlapping FTSE and S&P daily data the density function (and thus the likelihood function) does not simplify as it does in the usual contemporaneous case. The companion paper (Coleman 2007) discusses some tricks that allow one to derive a maximum likelihood and pseudo-maximum likelihood estimator. The pseudo-ML estimator combines the t_1-overlap and t_0-overlap data in an optimal manner but there is no simple solution to the first order conditions so that one must turn to numerical optimization techniques. In addition there is nothing that ensures the estimator $\rho \in [-1,1]$ so that constrained optimization must be used. This all makes the pseudo-ML estimator computationally difficult.

The pseudo-maximum likelihood estimator combines data optimally but is computationally intensive while the alternative MoM estimators are simple but may not be as efficient in all cases. A summary of the estimators I investigate is as follows:

- **Pseudo-ML Combined estimator** using the likelihood function (11) as for the ML Combined estimator but with complete (non-independent) t_1-overlap and t_0-overlap data. Using complete data has the benefit of not throwing out data.
- **Method-of-Moments Separate estimators** using complete (non-independent) t_1-overlap and t_0-overlap data separately to give two sets of estimators. This gives simple analytic formulae but again provides two estimators what will generally give different values.
- **Method-of-Moments (MoM) Combined estimator** using a weighted average of the method-of-moments separate estimators for the correlation coefficient.

Tables 1 through 5 below show simulation results comparing the three estimators: pseudo-ML combined, MoM separate, and MoM combined.[3] The simulations show the following:

Correlation estimators

The following observations are likely to be general, even though generalizing from a limited number of simulations is delicate:

- The pseudo-ML estimator uniformly performs as well or better (lower bias and standard error) than the alternate estimators considered. This is likely to be true more generally because the pseudo-ML estimator combines t_0-overlap and t_1-overlap data in a more efficient manner than alternate estimators. The better performance of the pseudo-ML estimator is more dramatic for $|\rho|>>0$.
- For $t_1 = t_0$ the MoM combined estimator performs as well as the pseudo-ML estimator. This is likely to be more general because when $t_1 = t_0$ the separate MoM estimators are, in a sense, balanced and it is reasonable to think that a simple average combines the information optimally.
- For $t_1 >> t_0$ the MoM t_1-separate estimator performs almost as well as the pseudo-ML estimator (standard errors almost as low), while the MoM combined estimator performs less well. As $t_0 \rightarrow 0$ this is not surprising since we move to the contemporaneous observations case. What is a little surprising is that this happens even for $t_1 = 0.8$.

[3] Results for the naïve estimator are not shown. The naïve estimator is biased and converges to $t_i \cdot \rho$ and the simulations reflect that fact. Each simulation is the result of 10,000.

- All estimators appear biased towards zero with the bias greater for larger ρ, smaller number of observations, and balanced data ($t_1 = t_0$).

The MoM estimators are less computationally burdensome than the pseudo-ML estimator but perform almost as well in certain cases, and it would be useful to have a rule specifying when to use the combined versus separate estimator. A simple, though ad-hoc rule, is to use the MoM combined or separate estimator depending on which has the lower standard error relative to the contemporaneous observation case. The following section develops some approximate formulae for the ratio of non-contemporaneous to contemporaneous standard errors (equations 13b and 13c), and these formulae can be used to implement such a rule. Note, however, that the pseudo-ML estimator should always have lower standard error than the MoM estimators under the maintained distributional assumptions.

Standard Errors - Variances

Standard errors for variances should be the same as in the contemporaneous case (accounting for reduced number of observations for ML estimators) since the variances depend on each series independently. The simulations bear this out.

Standard Errors - Covariance

The standard errors for non-contemporaneous estimators should be larger than for contemporaneous estimators. The simulations show that they are but not dramatically so. Tables 1 through 5 show the covariance estimator for the pseudo-ML combined estimators, together with the sample standard error and asymptotic standard error for the contemporaneous case (labeled "Theor SE"). The sample standard errors are between 1.1x and 1.8x the contemporaneous-case asymptotic standard errors. For example, table 1 shows that with ρ=0.8 and 480 observations the sample standard error is 0.0060 or 1.67x the asymptotic standard error for a contemporaneous ML estimator of 0.0036. When the overlap is not balanced (table 2 with t_0=0.2) the sample standard error is 0.0042 or 1.18x the asymptotic or theoretical value of 0.0036.

In the companion paper (Coleman 2007) I provide some heuristic arguments to derive an approximate expression for the ratio of the non-contemporaneous to the contemporaneous-case asymptotic sample standard error:

(13a) $$\sqrt{\left(2\frac{t_0 t_1}{t_1^2}+\frac{t_0^2}{t_1^2}\right)\frac{1}{\left(1+\rho^2\right)}+1}$$

For t_0=t_1=0.5 this will vary from 2.0 to 1.6 as ρ varies from 0 to 1, while for t_1=0.8 it varies from 1.25 to 1.13. I do not report the covariances for the separate estimators in the following tables, but examination of the detailed simulations shows that they match expression (13a) well.

One important point is that although the standard errors for the covariance in the non-contemporaneous case are larger than for the contemporaneous case, they are not dramatically so.

Standard Errors – Correlation

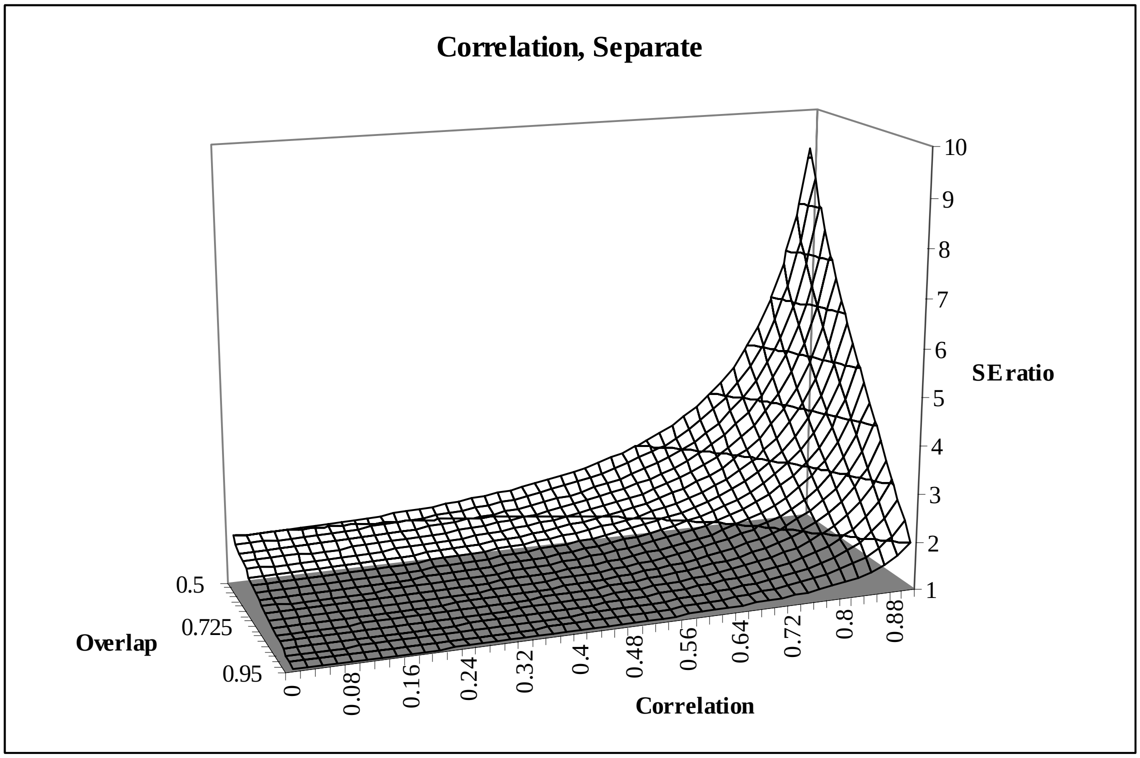
In contrast to the covariance, the standard errors for correlation estimators are dramatically larger than for the contemporaneous observations. Examination shows that they range from roughly 1.3x the contemporaneous case up to 4x the contemporaneous case or more.

In the companion paper I provide some heuristic arguments to derive an approximate expression for the ratio of the non-contemporaneous to the contemporaneous-case asymptotic sample standard error:

$$\sqrt{\left(2\frac{t_0 t_1}{t_1^2}+\frac{t_0^2}{t_1^2}\right)\cdot\frac{1}{\left(1-\rho^2\right)^2}+1} \tag{13b}$$

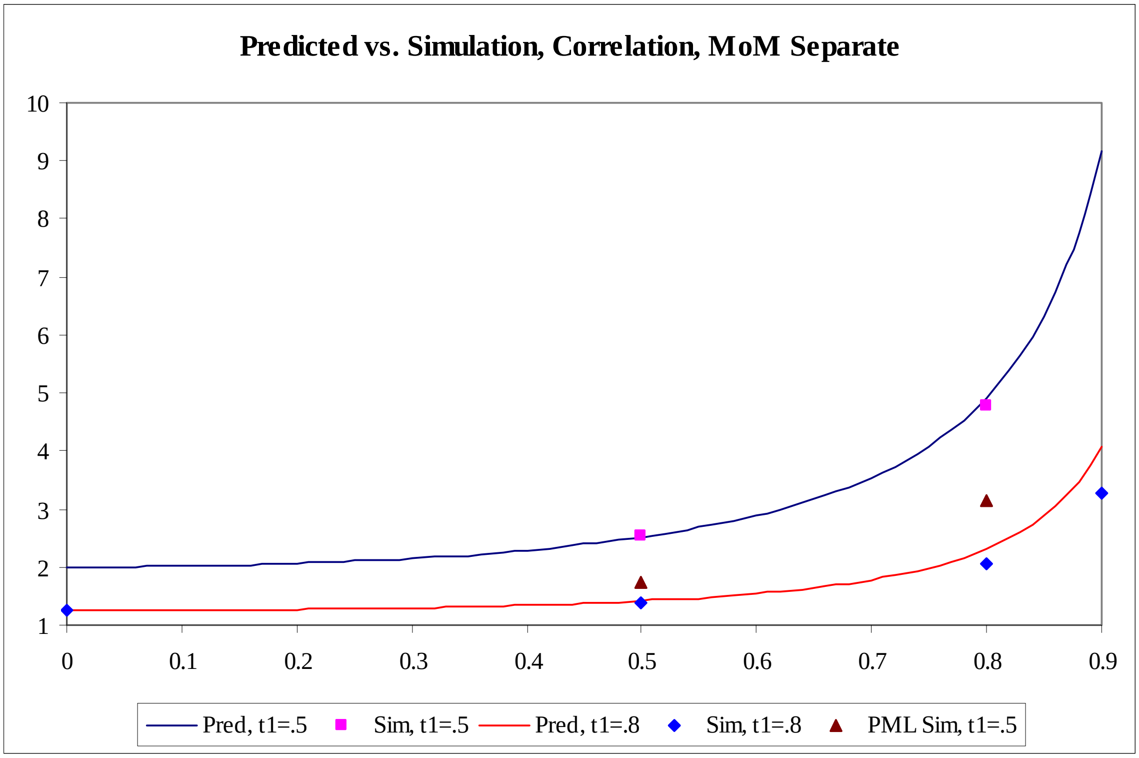
Although this expression is not derived specifically for the MoM separate estimators, it appears to apply reasonably well. The first term is the contribution from the "noise" of the independent normals. For $|\rho|>>0$ this can be very large relative to the second term. The best case (lowest contribution from the "noise") will be for $\rho=0$. For $t_0=t_1=0.5$ (13b) ranges from 2.0 to 4.9 as ρ ranges from 0 to 0.8, and increases without limit at ρ tends towards ± 1. For $t_0=0.2$ the expression ranges from 1.25 to 2.3 and up to ∞ as ρ goes from 0 to 0.8 and toward 1. Figure 3 shows expression (13b) for correlation between 0 and 0.9 and for overlap (t_1) between 0.95 and 0.5.

Figure 3 – Ratio of Non-contemporaneous to Contemporaneous Standard Error for Correlation Separate Estimator, Predicted by Expression (13b)



The simulations show that this expression is roughly correct. In table 1, for example, with ρ=0.8, t_0=t_1, and 480 observations the rho1-separate estimator has a sample standard error of0.079, 4.8x the asymptotic standard error of0.016, versus the above expression which predicts 4.9x. In table 3 with ρ=0.0, t_0=0.2, and 480 observations the rho1-separate estimator has a sample standard error of0.057, 1.25x the asymptotic value, the same as predicted by the above expression. Figure 4 shows the two slices through figure 1 corresponding to overlaps t_1=0.5 and t_1=0.8, together with the simulation results for MoM separate estimators. From this one can see that expression (13b) matches the simulation results quite well. We have also included the pseudo-ML simulation results. For equal overlap (t_1=t_0=0.5) they are below the ML separate (for ρ=0.8, 3.1 versus 4.6) while for t_1=0.8 they are indistinguishable from the corresponding results for the separate estimators.

Figure 4 – Ratio of Non-contemporaneous to Contemporaneous Standard Error for Separate Correlation Estimator, Predicted by Expression (13b) and Simulation for ML



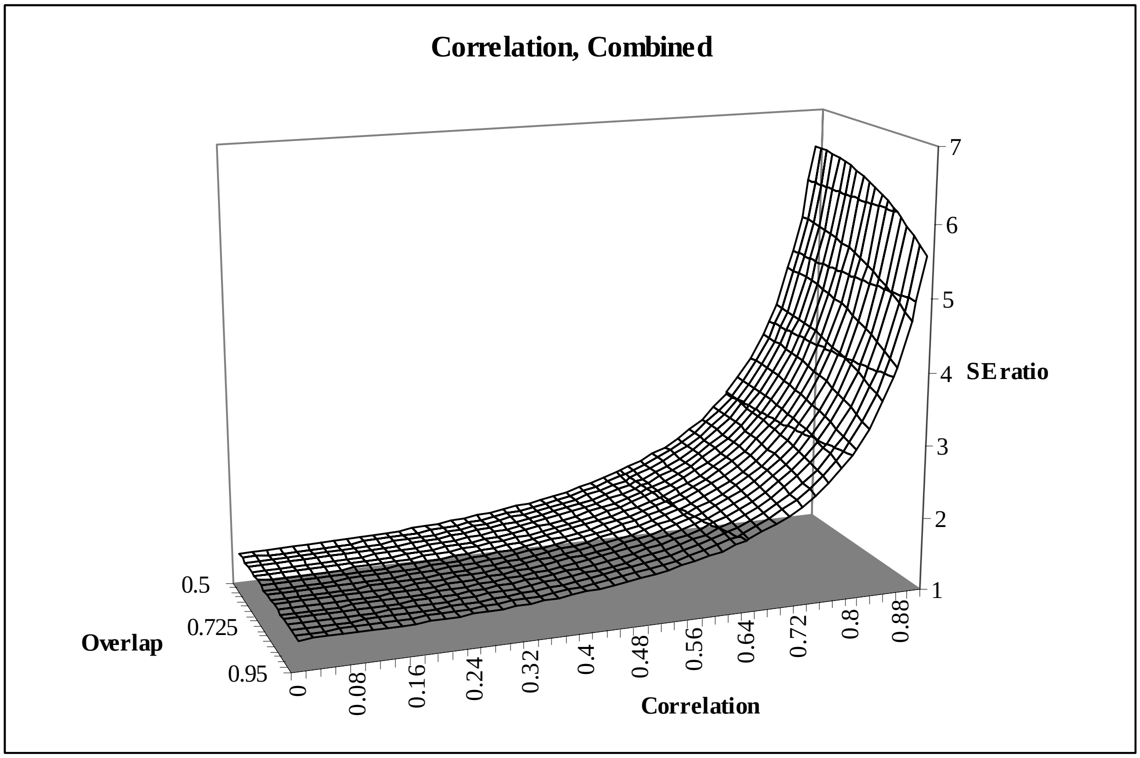
The conclusion is that for the correlation estimator, the standard errors can be substantially larger than for the contemporaneous case, and particularly when $|\rho|>>0$. Expressions (13a) and (13b) show that the non-overlapping "noise" (x^0 and y^0) substantially affects the precision of the correlation estimator when $|\rho|>>0$ but much less so the covariance estimator. They allow us to draw some tentative conclusions about how the precision of the separate estimators is likely to vary as the degree of overlap and the underlying correlation varies.

For the MoM combined estimator expression (13b) for t_1-overlap and t_0-overlap can be combined to give:

(13c) $$\sqrt{t_1^2 \cdot \left[\left(2\frac{t_0 t_1}{t_1^2} + \frac{t_0^2}{t_1^2}\right) \cdot \frac{1}{\left(1-\rho^2\right)^2} + 1\right] + t_0^2 \left[\left(2\frac{t_0 t_1}{t_0^2} + \frac{t_1^2}{t_0^2}\right) \cdot \frac{1}{\left(1-\rho^2\right)^2} + 1\right]}$$

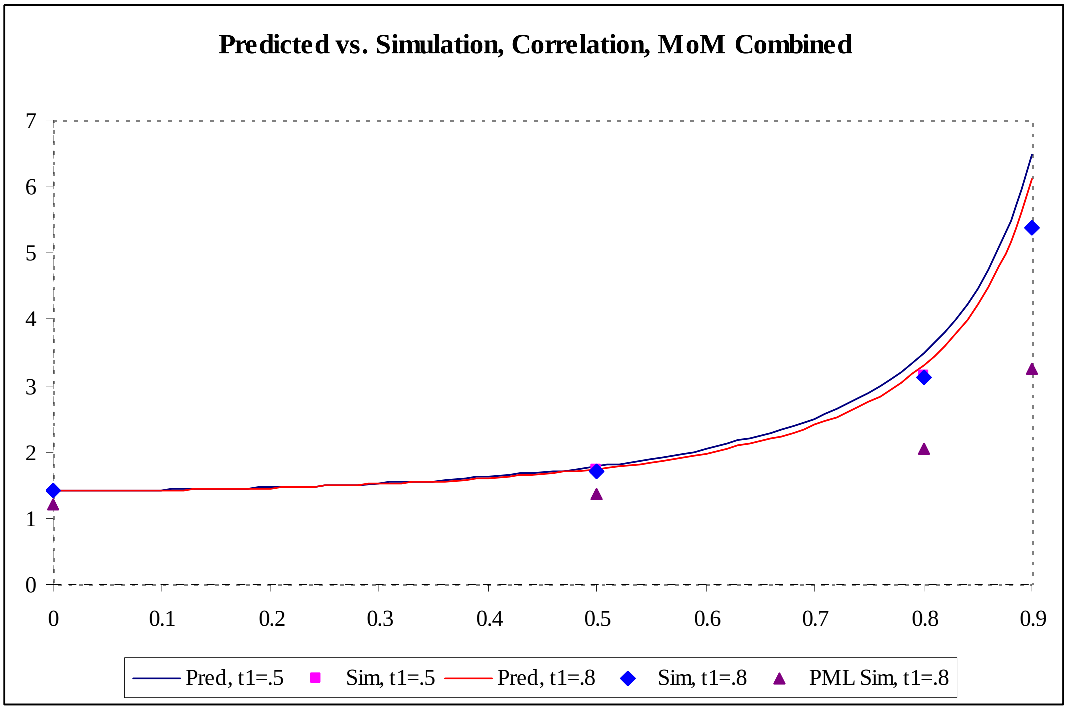
This expression is shown in Figure 5 for correlation between 0 and 0.9 and for overlap (t_1) between 0.95 and 0.5. Note that in contrast to expression (13b) for the separate estimator the SE ratio is much better behaved for equal overlap (t_1=t_0=0.5) but much worse behaved for large overlap (e.g. for t_1=0.8).

Figure 5 – Ratio of Non-contemporaneous to Contemporaneous Standard Error for Combined Correlation Estimator, Predicted by Expression (13c)



Expression (13c) matches the simulations for the MoM combined estimator pretty well, as shown in figure 6, which shows two slices through the surface. For t_1=0.5 and t_1=0.8 the simulation results for the MoM combined estimator are virtually the same and cannot be distinguished in the graphic. Figure 6 also includes the simulation results for the pseudo-ML estimator for t_1=0.8, where the standard error is lower than for the MoM combined. (For equal overlap, t_1=0.5, the pseudo-ML is indistinguishable from the MoM combined.)

Figure 6 – Ratio of Non-contemporaneous to Contemporaneous Standard Error for Correlation MoM Combined Estimator, Predicted by Expression (13b) and Simulation for Pseudo-ML



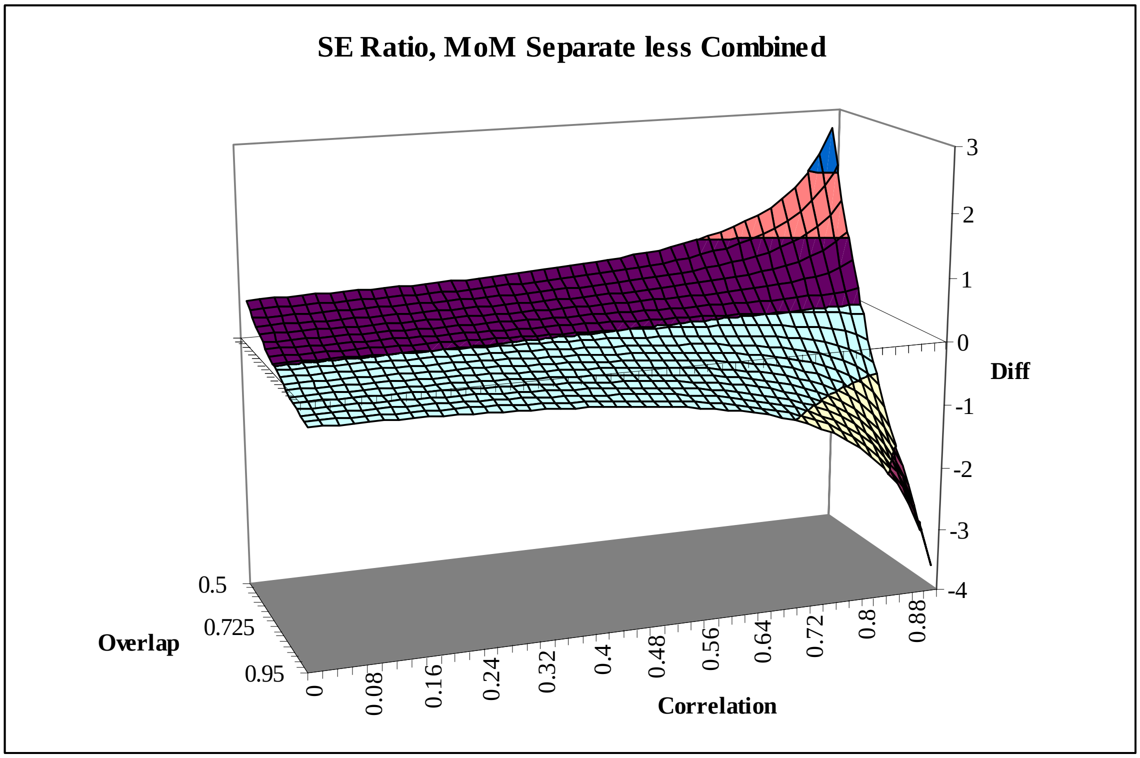
The conclusion seems to be:

- In all cases the pseudo-ML estimator has the lowest standard error relative to the contemporaneous-observation case (versus the MoM separate or the MoM combined estimators)
- For large overlap (e.g. t_1=0.8) the MoM separate estimator performs as well as the pseudo-ML estimator and substantially better than the MoM combined estimator
- For equal overlap the reverse is true, with the MoM combined estimator performing as well as the pseudo-ML estimator and substantially better than the MoM separate estimator.

This suggests a possible ad-hoc strategy using the MoM estimators rather than the more computationally intensive pseudo-ML estimator: Choose the MoM separate versus combined estimator based on expressions (13b) and (13c). If (13b) is lower this indicates the MoM separate estimator likely has lower standard error relative and vice versa. Figure 7 shows the difference between (13b) and (13c): a negative number indicates the MoM separate estimator likely has lower standard error. In practice we might use the separate estimator for $t_1 \approx> 0.70$ and the combined estimator $0.35 \approx< t_1 \approx< 0.65$. This proposed strategy is ad-hoc for two reasons. First, expressions (13b) and (13c) are only approximate and heuristic, although the simulations show they work reasonably well in specific cases. Second, there is some dependence on the correlation itself: at ρ=0.2 the cross-over point between (13b) and (13c) is t_1=0.70 while at ρ=0.8 it is at t_1=0.65. Note additionally that the MoM estimators will not always perform as well as the pseudo-

ML estimator when the overlap is intermediate between large ($t_1 \to 1$) and equal ($t_1 = t_0 = 0.5$).

Figure 7 – Difference Between Standard Error Ratio for Separate vs. Combined Estimators (Expression 13b less expression 13c)



For the maximum likelihood combined estimator we turn to the standard asymptotic result that the standard error will equal (the square root of) the diagonal of the inverse of the information matrix. We assume that this might also apply for the pseudo-ML estimator, and the simulations show that this is at least reasonable. In table 1 for $t_1 = t_0$ and 480 observations the standard error from the simulation is 0.052 versus 0.054 for the average of the asymptotic ML standard errors (diagonals of the information matrix). When the degree of overlap changes to $t_1 = 0.8$ as in table 2 the simulation for 480 observations gives sample standard error of 0.034 versus average of asymptotic ML errors of 0.031.

Estimated Correlations for S&P 500, FTSE 100, Nikkei 225

We apply the naïve, method-of-moments separate, simple MoM combined, and pseudo-ML combined estimators to data for the S&P 500, FTSE 100, and Nikkei 225 stock indexes. We use daily data covering the period 6 July 2006 through 16 May 2007, a total of 448 observations. The following three diagrams show the overlap for the three pairs of series, given that the Nikkei closes at 4pm Tokyo time, the FTSE at 4pm London time, and the S&P at 4pm New York time.

Overlap for FTSE 100 and S&P 500 Stock Indexes
```
 M11a        M4p          T11a          T4p
 |-- t0=5hr --|-- t1=19hr --|-- t0=5hr --|-- t1=19hr --|
   <-----    FTSE    ----->   <-----    FTSE    ----->
          <-----    S&P    ----->     <-----    S&P
```
t0 = 0.20833, t1 = 0.791667

Overlap for Nikkei 225 and S&P 500 Stock Indexes
```
 M3a          M4p           T3a           T4p
 |-- t0=13hr --|-- t1=11hr --|-- t0=13hr --|-- t1=11hr --|
    <-----    NKY    ----->     <-----    NKY    ----->
                <-----    S&P    ----->     <-----    S&P
```
t0 = 0.541667, t1 = 0.458333

Overlap for Nikkei 225 and FTSE 100 Stock Indexes
```
 M3a        M11a           T3a          T11a
 |-- t0=8hr --|-- t1=16hr --|-- t0=8hr --|-- t1=16hr --|
   <-----    NKY    ----->     <-----    NKY    ----->
                <-----   FTSE    ----->     <-----   FTSE
```
t0 = 0.33333, t1 = 0.66667

When using actual data one immediately encounters the problem of holidays and weekends. For now we pretend that all business days are the same, but a possible relaxation of this assumption is discussed in the next section.

Table 5 shows the various estimates. Focusing for now on the S&P and FTSE, the naïve estimator (ignoring that data are non-contemporaneous) gives a correlation of 0.439. The MoM separate estimator (simply adjusting for the length of the t_1-overlap) raises the estimate to 0.555. Using the t_0-overlap data gives an estimate of 1.176, above 1.[4] The pseudo-ML estimator is close to the t_1-overlap estimator at 0.576 while the MoM combined estimator is higher at 0.684. In this case the simple ad-hoc rule – use the MoM separate t_1-estimator for $t_1 \approx> 0.70$ – would lead to using the separate estimator and an answer close to the pseudo-ML estimate.[5]

[4] The pattern that the MoM separate t_0-overlap estimator is higher than the t1-overlap estimator is consistent for all three pairs. This may be a result of assuming that price volatility is homogeneous over time, whereas in reality more events may occur during hours when markets are open. A possible relaxation of that assumption is discussed in the next section.

[5] The naïve correlation between S&P and lagged FTSE is -0.05. This is consistent with serial independence of returns since S&P today returns have no overlap with yesterday FTSE returns. For FTSE vs. lagged Nikkei it is -0.04 and for S&P vs. lagged Nikkei it is -0.07.

Table 5 – Various Estimates of Correlation Across Stock Indexes

	FTSE / SPX	NKY / FTSE	NKY / SPX
t1 – length of "same-day" overlap	0.792	0.667	0.458
t0 – length of "next-day" overlap	0.208	0.333	0.542
Naive estimator	0.439	0.333	0.133
Meth-of-Mom separate, t1 data	0.555	0.499	0.291
Meth-of-Mom separate, t0 data	**1.176**	0.718	0.613
Meth-of-Mom combined	0.684	0.572	0.465
Pseudo-ML combined	0.576	0.533	0.484
asymptotic SE	0.043	0.055	0.063

For these three pairs of series the ad-hoc rule discussed in section 8 above seems to work, in giving results reasonably close to the pseudo-ML results. For FTSE/SPX the overlap is large (t_1=0.79) and the estimates from pseudo-ML and MoM separate are reasonably close. For NKY/SPX the overlap is almost equal (t_1=0.46) and the pseudo-ML and MoM combined estimates are close. The NKY/FTSE is intermediate with the rule implying neither estimator is clearly preferred, and in this case neither estimator is particularly close to the pseudo-ML estimate.

Conclusions and Extensions

This paper has discussed various estimators for the correlation when observations are partially overlapping but not fully contemporaneous. Via simulations I have investigated some of the estimators, together with their standard errors. From the simulations we conclude that the pseudo-ML estimator works best. I have proposed a simple, though somewhat ad-hoc rule to use the computationally-simpler MoM estimators: use the separate estimator when the overlap $t_1 \approx> 0.7$ (or $t_0 \approx> 0.7$) and use the MoM combined estimator when $0.35 \approx< t_1 \approx< 0.65$.[6] In all cases careful attention should be paid to the standard errors since they will be larger, sometimes substantially larger, than in the usual normal case. Expressions (13b) and (13c) can be used to roughly estimate the non-contemporaneous standard errors relative to the contemporaneous standard errors.

[6] Note that the good performance of the MoM separate estimator for large overlap implies that for weekly data, where t_1>0.93, simply adjusting the naïve correlation by the degree of overlap (i.e. using the MoM separate estimator) should work very well.

Table 1 – Simulation Results for 10,000 Trials – σ_{xx}=0.0400, σ_{yy}=0.0625, ρ=0.80, t1=t0=1/2

	Pseudo-ML - full data				Method-of-Moments			
	Mean	StdErr	Theor SE	ML SEs	Mean	StdErr	Theor SE	% not [-1,+1]
30 obs								
rho: t1-separate					0.749	0.261	0.066	27.0%
rho: t0-separate					0.746	0.263	0.066	26.8%
rho combined	**0.776**	**0.183**	**0.066**	**0.218**	**0.776**	**0.183**		15.4%
variance ser 1	0.0398	0.0101	0.0102	0.0072				
variance ser 2	0.0622	0.0160	0.0159	0.0113				
covar comb	0.0389	0.0141	0.0117					
480 obs								
rho: t1-separate					0.799	0.079	0.016	0.4%
rho: t0-separate					0.799	0.078	0.016	0.4%
rho comb	**0.799**	**0.052**	**0.016**	**0.054**	**0.799**	**0.052**		
variance ser 1	0.0400	0.0026	0.0026	0.0018				
variance ser 2	0.0625	0.0040	0.0040	0.0029				
covar comb	0.0400	0.0040	0.0029					

"Theor SE" is the (asymptotic) standard error for the contemporaneous case, with formulae given in the text

Table 2 – Simulation Results for 10,000 Trials – σ_{xx}=0.0400, σ_{yy}=0.0625, ρ=0.80, t1=4/5

	Pseudo-ML - full data				Method-of-Moments			
	Mean	StdErr	Theor SE	ML SEs	Mean	StdErr	Theor SE	% not [-1,+1]
30 obs								
rho: t1-separate					0.793	0.139	0.066	4.0%
rho: t0-separate					0.530	0.608	0.066	45.0%
rho combined	**0.794**	**0.135**	**0.066**	**0.122**	**0.779**	**0.180**		15.7%
variance ser 1	0.0401	0.0102	0.0102	0.0072				
variance ser 2	0.0624	0.0159	0.0159	0.0112				
covar comb	0.0401	0.0131	0.0117					
480 obs								
rho: t1-separate					0.799	0.034	0.016	
rho: t0-separate					0.774	0.193	0.016	18.7%
rho comb	**0.799**	**0.034**	**0.016**	**0.031**	**0.799**	**0.051**		
variance ser 1	0.0400	0.0025	0.0026	0.0018				
variance ser 2	0.0625	0.0040	0.0040	0.0028				
covar comb	0.0400	0.0033	0.0029					

For 30 obs, 3 simulations exited from optimization routine before converging, 1 simulations gave non-pd jacobian
"Theor SE" is the (asymptotic) standard error for the contemporaneous case, with formulae given in the text

Table 3 – Simulation Results for 10,000 Trials – σ_{xx}=0.0400, σ_{yy}=0.0625, ρ=0.0, t1=4/5

	Pseudo-ML - full data				Method-of-Moments			
	Mean	StdErr	Theor SE	ML SEs	Mean	StdErr	Theor SE	% not [-1,+1]
30 obs								
rho: t1-separate					0.002	0.227	0.183	
rho: t0-separate					-0.068	0.816	0.183	28.3%
rho combined	**0.002**	**0.221**	**0.183**	**0.214**	**0.000**	**0.259**		0.0%
variance ser 1	0.0401	0.0102	0.0102	0.0073				
variance ser 2	0.0624	0.0161	0.0159	0.0114				
covar comb	0.0001	0.0111	0.0091					
480 obs								
rho: t1-separate					0.000	0.057	0.046	
rho: t0-separate					0.000	0.226	0.046	
rho comb	**0.000**	**0.055**	**0.046**	**0.055**	**0.000**	**0.065**		
variance ser 1	0.0400	0.0026	0.0026	0.0018				
variance ser 2	0.0624	0.0040	0.0040	0.0028				
covar comb	0.0000	0.0028	0.0023					

For 30 obs, 38 simulations exited from optimization routine before converging, 0 simulations gave non-pd jacobian

"Theor SE" is the (asymptotic) standard error for the contemporaneous case, with formulae given in the text

Table 4 – Simulation Results for 10,000 Trials – σ_{xx}=0.0400, σ_{yy}=0.0625, ρ=0.5, t1=1/2

	Pseudo-ML - full data				Method-of-Moments			
	Mean	StdErr	Theor SE	ML SEs	Mean	StdErr	Theor SE	% not [-1,+1]
30 obs								
rho: t1-separate					0.488	0.332	0.137	6.6%
rho: t0-separate					0.485	0.330	0.137	6.5%
rho combined	**0.493**	**0.234**	**0.137**	**0.239**	**0.493**	**0.234**		1.1%
variance ser 1	0.0400	0.0102	0.0102	0.0073				
variance ser 2	0.0625	0.0163	0.0159	0.0114				
covar comb	0.0250	0.0140	0.0102					
480 obs								
rho: t1-separate					0.499	0.087	0.034	
rho: t0-separate					0.499	0.086	0.034	
rho comb	**0.499**	**0.059**	**0.034**	**0.060**	**0.499**	**0.059**		
variance ser 1	0.0400	0.0026	0.0026	0.0018				
variance ser 2	0.0625	0.0040	0.0040	0.0029				
covar comb	0.0250	0.0035	0.0026					

"Theor SE" is the (asymptotic) standard error for the contemporaneous case, with formulae given in the text

Table 5 – Simulation Results for 10,000 Trials – σ_{xx}=0.0400, σ_{yy}=0.0625, ρ=0.5, t1=4/5

	Pseudo-ML - full data				Method-of-Moments			
	Mean	StdErr	Theor SE	ML SEs	Mean	StdErr	Theor SE	% not [-1,+1]
30 obs								
rho: t1-separate					0.495	0.195	0.137	0.1%
rho: t0-separate					0.328	0.698	0.137	34.6%
rho combined	**0.495**	**0.190**	**0.137**	**0.177**	**0.493**	**0.231**		1.0%
variance ser 1	0.0400	0.0102	0.0102	0.0073				
variance ser 2	0.0624	0.0163	0.0159	0.0113				
covar comb	0.0251	0.0122	0.0102					
480 obs								
rho: t1-separate					0.499	0.048	0.034	
rho: t0-separate					0.498	0.225	0.034	1.4%
rho comb	**0.500**	**0.047**	**0.034**	**0.045**	**0.499**	**0.058**		
variance ser 1	0.0400	0.0026	0.0026	0.0018				
variance ser 2	0.0625	0.0040	0.0040	0.0028				
covar comb	0.0250	0.0030	0.0026					

For 30 obs, 19 simulations exited from optimization routine before converging

"Theor SE" is the (asymptotic) standard error for the contemporaneous case, with formulae given in the text

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