TMG Financial Products Inc.
475 Steamboat Road
Greenwich, CT 06830

Thomas S. Coleman
VP, Risk Management
203-861-8993

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CONSTANT MATURITY SWAPS AND ADJUSTING FOR CONVEXITY

A constant maturity swap is a variation on a standard basis swap. One side is LIBOR as usual, but the other side is determined using a rate such as the 5 year swap rate or the 5 year Treasury rate. Constant maturity swaps can use a variety of indexes. The Federal Reserve's constant maturity Treasury (CMT) index is the most common, with constant maturity swap (CMS) rates being the next most common.

CMT/CMS swaps can be used to adjust duration (for example receiving the 10 year CMT rate on a swap can help reduce a mortgage portfolio's exposure to the 10 year sector) or to take a view on the shape of the curve (receiving the 5 year CMS rate and paying LIBOR would benefit from a widening on the spread between 3 month LIBOR and 5 year swap rates).

A CMS/CMT swap trades at a spread to floating LIBOR. The spread is a result of:

1. CURVE: If the curve is upward sloping, the CMS/CMT rate will be higher than the LIBOR rate and, to receive the CMS/CMT rate, one must pay a spread over LIBOR.
2. DAY COUNT BASIS: The CMS/CMT side often pays quarterly but uses a semi-annually quoted rate. (E.g., a 7%sab rate converts to a 6.94%qb rate: receiving a 7%sab rate quarterly is a benefit of about 6bp.)
3. CONVEXITY: The linearity of the CMS/CMT payment combined with the convexity of hedge instruments leads to a benefit to receiving CMS/CMT: one must pay a spread over LIBOR to receive the CMS/CMT rate.

The first two effects are straight-forward, but the convexity adjustment is more difficult.

The convexity adjustment arises because the CMS/CMT payment is linear in the index while the hedge, a standard swap, is convex. Consider a single reset on the CMS leg of a swap receiving the 10 year CMS rate. If the CMS rate changes by 1bp the profit or loss is 1¢.[1] To hedge this, one would receive fixed on $14.39 notional of a 10 year swap: when the 10 year swap rate changes by 1bp the profit or loss would also be 1¢.

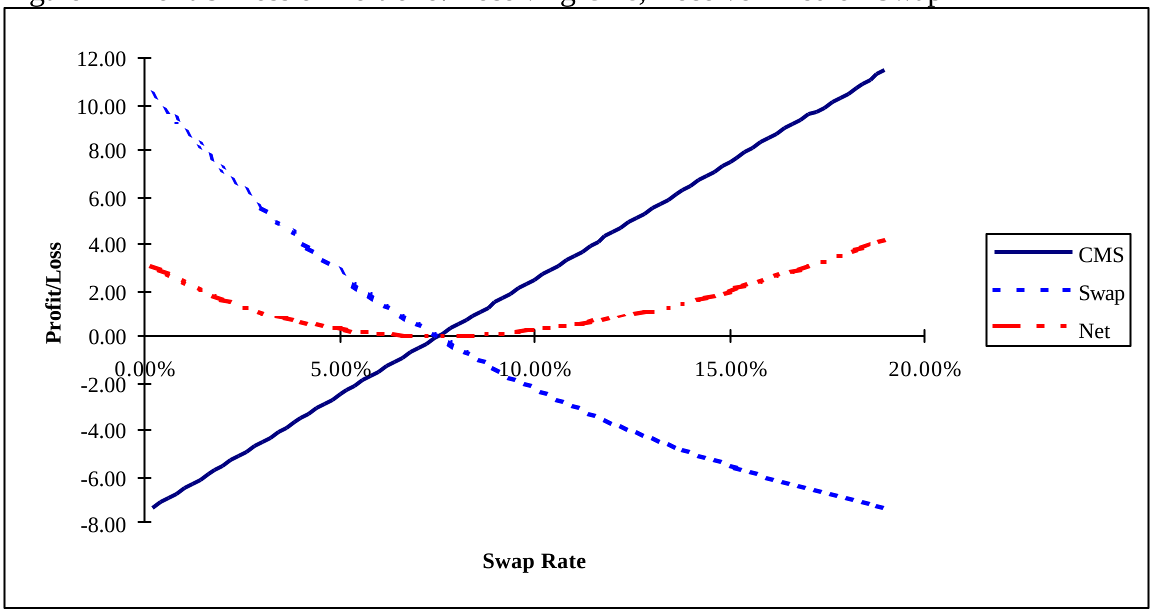
The convexity effect arises when the change is larger than 1bp. If the CMS index rises by 100bp the profit on the CMS leg is $1. The hedging swap, however, would lose $0.96. If the CMS index falls by 100bp the loss on the CMS leg is $1 while the profit on the swap would be $1.05.

[1] This assumes a flat 7.5%sab curve and a notional of $100 on the CMS swap.

Figure 1 shows the hedge mismatch graphically. The CMS leg is linear in the swap rate while the hedging swap is convex.

Figure 1 also shows the net profit and loss of a portfolio of receiving the CMS rate and hedging by receiving fixed on a standard swap. The profit from this strategy is always non-negative; it is impossible to lose money by receiving the CMS rate and hedging with a standard swap. Given that there are few (if any) free lunches in the financial markets, this is unlikely. The result is that one receives the CMS rate **less** a spread (over-and-above the spread necessary to bring the CMS rate back to LIBOR). This spread is the convexity spread.

Figure 1 - Profit & Loss on Portfolio: Receiving CMS, Receive Fixed on Swap



The size of the convexity spread will depend on three variables:

- Time to reset: the longer the more chance that the swap rate moves up or down.
- Volatility of the index (the forward swap rate): the higher the volatility the more the rate may move in a given period of time.
- The length of the underlying forward swap rate: the longer the hedging swap, the greater the convexity effect.

It is easy to explain how and why the convexity spread arises, but more difficult to calculate exactly how much it should be. One simple approach is what we might call the equivalent martingale method. The convexity adjustment must be calculated separetely for each reset of a CMS/CMT swap. For a single reset, model today's value of the hedging swap as the average of the future values (appropriately discounted). The future values are themselves modeled as a function of the future swap rates, which are assumed to be log-normally distributed. Thus, today's value is modeled as the average future value, where the average is taken over a log-normal distribution of swap rates. The problem is choosing the mean of the rate distribution.

Because the the swap value is a convex function of the swap rate, if the mean of the distribution were equal to the forward swap rate the expected value of the hedging swap would not equal the actual value. To make the expected value equal to the actual value, one must adjust the mean upwards. Generally, solving for the adjusted mean involves finding the zero of an integral equation. There are some tricks, however, which make this quite fast.

The resulting distribution of future swap rates, using the adjusted mean, is the equivalent martingale measure. Consider using this measure to value the CMS/CMT payment. Since this payment is linear, the expected value is simply the mean of the distribution. In other words, the adjusted mean under the equivalent martingale measure is the convexity adjusted CMS/CMT rate. The difference between the adjusted mean and the forward swap rate is the convexity spread.

An example may help. Consider the 100bp movements considered above, and suppose the "model" is that rates move up 100bp or down 100bp with equal probability from a mean of 7.5%. The value of the swap would be

$$½*(1.05) + ½*(-0.96) = 0.045$$

i.e., too high by 0.045. To make the value equal to the true value (zero), the mean must be adjusted upwards from 7.5% to 7.545%. The expected value would then equal zero.

The convexity effect can be sizable. For a 10 year swap against five year CMS/CMT the adjusted mean swap rate is 34bp above the forward rate for the last reset (at 9.75 years). For the whole swap, the convexity spread is about 14bp (over and above any spread resulting from slope of the curve and day basis). For a 10 year swap against 10 year CMS/CMT, the overall convexity spread is about 26bp.[2]

The same convexity adjustment and convexity effects that arise in CMS/CMT swaps also arise in CMS/CMT caps and floors, and LIBOR-in-arrears basis swaps. The equivalent martingale approach can also be used for these instruments.

[2] Assuming a flat 7.5%sab curve, quarterly resets and payments, and 15% volatility.